Key

Math 4

Name ____________________

Date________

4-6 Practice

I can, without a calculator, use trigonometric identities such as angle addition/subtraction and double angle formulas, to express values of trigonometric functions in terms of rational numbers and radicals.

SHOW ALL WORK. EXACT VALUES ONLY. NO CALCULATOR!!!!

Simplify the following:

1. $\cos\frac{\pi}{8}\cos\frac{7\pi}{8}+\sin\frac{\pi}{8}\sin\frac{7\pi}{8}=$

$= \cos\left(\frac{\pi}{8}-\frac{7\pi}{8}\right)$

$= \cos\left(\frac{-6\pi}{8}\right)$

$= \cos\left(-\frac{3\pi}{4}\right) = \boxed{-\frac{\sqrt{2}}{2}}$

2. $\cos(-5^\circ)\cos 245^\circ - \sin(-5^\circ)\sin 245^\circ =$

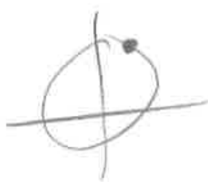
$= \cos(-5+245)$

$= \cos(240^\circ)$

$= \boxed{-\frac{1}{2}}$

3. $\cos\frac{5\pi}{9}\cos\frac{2\pi}{9}+\sin\frac{5\pi}{9}\sin\frac{2\pi}{9}=$

$= \cos\left(\frac{5\pi}{9}-\frac{2\pi}{9}\right)$



$= \cos\left(\frac{3\pi}{9}\right)$

$= \cos\left(\frac{\pi}{3}\right)$

$= \boxed{\frac{1}{2}}$

4. $\cos(15^\circ)=$

$= \cos(60^\circ - 45^\circ)$

$= \cos 60^\circ \cdot \cos 45^\circ + \sin 60^\circ \cdot \sin 45^\circ$

$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

5. $\cos\left(\frac{13\pi}{12}\right)=$

$= \cos\left(\frac{10\pi}{12}+\frac{3\pi}{12}\right)$

$= \cos\left(\frac{5\pi}{6}+\frac{\pi}{4}\right)$

$= \cos\frac{5\pi}{6}\cdot\cos\frac{\pi}{4} - \sin\frac{5\pi}{6}\cdot\sin\frac{\pi}{4}$

$= -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \boxed{-\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}}$

6. $\cos(255^\circ)=$

$= \cos(210+45)$

$= \cos 210 \cdot \cos 45 - \sin 210 \cdot \sin 45$

$= -\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} - -\frac{1}{2}\cdot\frac{\sqrt{2}}{2}$

$= -\frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4}$

$= \boxed{\frac{\sqrt{2}-\sqrt{6}}{4} \text{ or } \frac{-\sqrt{6}+\sqrt{2}}{4}}$

More Challenging Problems

7. $\cos(\alpha+\beta)-\cos(\alpha-\beta)=$

$= \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta - (\cos\alpha\cdot\cos\beta + \sin\alpha\cdot\sin\beta)$

$= \cancel{\cos\alpha\cdot\cos\beta} - \sin\alpha\cdot\sin\beta - \cancel{\cos\alpha\cdot\cos\beta} - \sin\alpha\cdot\sin\beta$

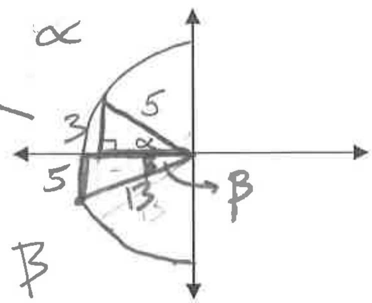
$= \boxed{-2\sin\alpha\cdot\sin\beta}$

Adjacent

8. Given: $\cos\alpha = -\frac{4}{5},\ \frac{\pi}{2} < \alpha < \pi$ (hypotenuse: 5)

$\sin\beta = -\frac{5}{13},\ \pi < \beta < \frac{3\pi}{2}$ (5 opp., 13 Hyp.)

$\sin\alpha = \frac{3}{5}$



Find: $\cos(\alpha + \beta)$

$= \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$

$= -\frac{4}{5} \cdot -\frac{12}{13} - \frac{3}{5} \cdot -\frac{5}{13}$

$= \frac{48}{65} + \frac{15}{65}$

$= \boxed{\frac{63}{65}}$

$5^2 + b^2 = 13^2$

$25 + b^2 = 169$

$b^2 = 144$

$b = 12$

So, $\cos\beta = -\frac{12}{13}$

9. Verify: $\cos\left(\frac{3\pi}{2} - x\right) = -\sin x$

$= \cos\frac{3\pi}{2} \cdot \cos x + \sin\frac{3\pi}{2} \cdot \sin x$

$= 0 \cdot \cos x - 1 \cdot \sin x$

$= \boxed{-\sin x}$ ✓

10. Verify: $\cos(\pi + x) = -\cos x$

$= \cos\pi \cdot \cos x - \sin\pi \cdot \sin x$

$= -1 \cdot \cos x - 0 \cdot \sin x$

$= \boxed{-\cos x}$ ✓

11. Verify: $\cos(\alpha + \beta)\cos(\alpha - \beta) = \cos^2\alpha - \sin^2\beta$

$= (\cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta)(\cos\alpha \cdot \cos\beta + \sin\alpha \cdot \sin\beta)$

$= \cos^2\alpha \cdot \cos^2\beta + \cancel{\cos\alpha \cos\beta \cdot \sin\alpha \cdot \sin\beta} - \cancel{\cos\alpha \cdot \cos\beta \cdot \sin\alpha \cdot \sin\beta} - \sin^2\alpha \cdot \sin^2\beta$

$= \cos^2\alpha \cdot \cos^2\beta - \sin^2\alpha \cdot \sin^2\beta$

$= \cos^2\alpha\,(1 - \sin^2\beta) - (1 - \cos^2\alpha) \cdot \sin^2\beta$

$= \cos^2\alpha - \cancel{\cos^2\alpha \sin^2\beta} - (\sin^2\beta - \cancel{\cos^2\alpha \cdot \sin^2\beta})$

$= \boxed{\cos^2\alpha - \sin^2\beta}$